Contents

1 Introduction **1**

2 Cross sections and nuclear data **5**
2.1 Solid angles and spherical harmonics . . . 5
2.2 Dealing with distributions . . . 9
2.3 Dynamics of a scattering reaction . . . 12
2.3.1 Collision of a neutron with a nucleus initially at rest . . . 16
2.4 Definition of a cross section . . . 19
2.5 Formation of a compound nucleus . . . 22
2.5.1 The single level Breit-Wigner formulas . . . 28
2.5.2 Low-energy variation of cross sections . . . 31
2.6 Thermal agitation of nuclides and binding effects . . . 33
2.6.1 Numerical convolution of cross sections . . . 34
2.6.2 Convolution of Breit-Wigner cross sections . . . 36
2.6.3 Convolution of a constant cross section . . . 39
2.6.4 Convolution of the differential scattering cross section . . . 41
2.6.5 Effects of molecular or metallic binding . . . 46
2.7 Expansion of the differential cross sections . . . 50
2.8 Calculation of the probability tables . . . 53
2.9 Production of an isotopic cross-section library . . . 54
2.9.1 Photo-atomic interaction data . . . 59
2.9.2 Delayed neutron data . . . 60
2.9.3 An overview of DRAGR . . . 61
Exercises . . . 64

3 The transport equation **69**
3.1 The particle flux . . . 69
3.2 Derivation of the transport equation . . . 72
3.2.1 The characteristic form of the transport equation . . . 74
3.2.2 The integral form of the transport equation . . . 75
3.2.3 Boundary and continuity conditions . . . 76
3.3 Source density in reactor physics . . . 77
3.3.1 The steady-state source density . . . 77
3.3.2 The transient source density . . . 80
3.4 The transport correction . . . 82
3.5 Multigroup discretization . . . 83
3.5.1 Multigroup steady-state transport equation . . . 85
3.5.2 Multigroup transient transport equation . . . 87
3.6 The first-order streaming operator . . . 89
3.6.1 Cartesian coordinate system . . . 89

3.6.2 Cylindrical coordinate system . . . 91
3.6.3 Spherical coordinate system . . . 93
3.7 The spherical harmonics method . . . 94
3.7.1 The P_n method in 1D slab geometry . . . 95
3.7.2 The P_n method in 1D cylindrical geometry . . . 99
3.7.3 The P_n method in 1D spherical geometry . . . 104
3.7.4 The simplified P_n method in 2D Cartesian geometry . . . 106
3.8 The collision probability method . . . 108
3.8.1 The interface current method . . . 110
3.8.2 Scattering-reduced matrices and power iteration . . . 113
3.8.3 Slab geometry . . . 113
3.8.4 Cylindrical 1D geometry . . . 118
3.8.5 Spherical 1D geometry . . . 125
3.8.6 Unstructured 2D finite geometry . . . 126
3.9 The discrete ordinates method . . . 133
3.9.1 Quadrature sets in the method of discrete ordinates . . . 133
3.9.2 The difference relations in 1D slab geometry . . . 138
3.9.3 The difference relations in 1D cylindrical geometry . . . 140
3.9.4 The difference relations in 1D spherical geometry . . . 144
3.9.5 The difference relations in 2D Cartesian geometry . . . 146
3.9.6 Synthetic acceleration . . . 148
3.10 The method of characteristics . . . 150
3.10.1 The MOC integration strategy . . . 151
3.10.2 Unstructured 2D finite geometry . . . 157
3.10.3 The algebraic collapsing acceleration . . . 162
3.11 The multigroup Monte Carlo method . . . 167
3.11.1 Mathematical background . . . 167
3.11.2 Rejection techniques . . . 172
3.11.3 The random walk of a neutron . . . 176
3.11.4 Criticality calculations . . . 184
3.11.5 Monte Carlo reaction estimators . . . 189
Exercises . . . 191

4 Elements of lattice calculation 195
4.1 A historical overview . . . 196
4.2 Neutron slowing-down and resonance self-shielding . . . 199
4.2.1 Elastic slowing down . . . 201
4.2.2 A review of resonance self-shielding approaches . . . 204
4.2.3 The Livolant-Jeanpierre approximations . . . 205
4.2.4 The physical probability tables . . . 209
4.2.5 The statistical subgroup equations . . . 212
4.2.6 The multigroup equivalence procedure . . . 215
4.3 The neutron leakage model . . . 217
4.3.1 The B_n leakage calculation . . . 218
4.3.2 The homogeneous fundamental mode . . . 219
4.3.3 The heterogeneous fundamental mode . . . 224
4.3.4 Introduction of leakage rates in a lattice calculation . . . 227
4.3.5 Introduction of leakage rates with collision probabilities . . . 229
4.3.6 Full-core calculations in diffusion theory . . . 231
4.3.7 Full-core calculations in transport theory . . . 232
4.4 The SPH equivalence technique . . . 234
4.4.1 Definition of the macro balance relations . . . 235

4.4.2 Definition of the SPH factors . . . 236
4.4.3 Iterative calculation of the SPH factors . . . 240
4.5 Isotopic depletion . . . 241
4.5.1 The power normalization . . . 244
4.5.2 The saturation model . . . 245
4.5.3 The integration factor method . . . 247
4.5.4 Depletion of heavy isotopes . . . 249
4.6 Creation of the reactor database . . . 252
4.6.1 Selected information . . . 252
4.6.2 Database information structure . . . 254
4.7 A presentation of DRAGON . . . 255
4.7.1 A DRAGON tutorial . . . 256
Exercises . . . 259

5 Full-core calculations **271**
5.1 The steady-state diffusion equation . . . 274
5.1.1 The Fick law . . . 274
5.1.2 Continuity and boundary conditions . . . 276
5.1.3 The finite homogeneous reactor . . . 278
5.1.4 The heterogeneous 1D slab reactor . . . 280
5.2 Discretization of the neutron diffusion equation . . . 284
5.2.1 Mesh-corner finite differences . . . 285
5.2.2 Mesh-centered finite differences . . . 288
5.2.3 A primal variational formulation . . . 290
5.2.4 The Lagrangian finite-element method . . . 292
5.2.5 The analytic nodal method in 2D Cartesian geometry . . . 296
5.3 Generalized perturbation theory . . . 303
5.3.1 Mathematical background . . . 303
5.3.2 State variables and reactor characteristics . . . 306
5.3.3 Computing the Jacobian using the implicit approach . . . 308
5.3.4 Computing the Jacobian using the explicit approach . . . 309
5.4 Space-time kinetics . . . 310
5.4.1 Point-kinetics equations . . . 311
5.4.2 The implicit temporal scheme . . . 316
5.4.3 The space-time implicit scheme . . . 318
Exercises . . . 321

Answers to Problems **327**

A Tracking of 1D and 2D geometries **335**
A.1 Tracking of 1D slab geometries . . . 335
A.2 Tracking of 1D cylindrical and spherical geometries . . . 337
A.3 The theory behind sybt1d . . . 340
A.4 Tracking of 2D square pincell geometries . . . 341
A.5 The theory behind sybt2d . . . 346

B Special functions with Matlab **349**
B.1 Function taben . . . 349
B.2 Function akin . . . 350

C Numerical methods **353**
C.1 Solution of a linear system . . . 353
C.1.1 Gauss elimination . . . 353
C.1.2 Cholesky factorization . . . 355
C.1.3 QR factorization . . . 358
C.1.4 Iterative approach . . . 360
C.2 Solution of an eigenvalue problem . . . 367
C.2.1 The inverse power method . . . 368
C.2.2 The inverse power method without inversion . . . 374
C.2.3 The preconditioned power method . . . 380
C.2.4 The Hotelling deflation . . . 381
C.2.5 The multigroup partitioning . . . 383
C.2.6 Convergence acceleration . . . 385
C.3 Solution of a fixed-source eigenvalue problem . . . 388
C.3.1 The inverse power method . . . 389
C.3.2 The preconditioned power method with variational acceleration . . . 391

Bibliography **394**

Index **405**